

Math 304 (Spring 2015) - Homework 2

Problem 1.

Suppose $A = \begin{pmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 1 & -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 4 & 1 \\ -2 & 3 & 1 \\ 2 & 0 & 4 \end{pmatrix}$. Compute

$$(a). A + 2B, \quad (b). AB, \quad (c). BA, \quad (d). (AB)^T.$$

Problem 2.

Given $A = \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$, compute A^2 , A^3 , A^4 and A^5 . Recall that for any natural number k ,

$$A^k = \underbrace{AA \cdots A}_{k \text{ times}}$$

Problem 3.

Determine whether each of the following matrices has an inverse or not. If yes, find the inverse.

$$(a) \begin{pmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{pmatrix}$$

$$(b) \begin{pmatrix} 1 & 3 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & -6 \end{pmatrix}$$

$$(c) \begin{pmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{pmatrix}$$

Problem 4.

Solve the following linear system

$$\begin{cases} x_1 + x_3 = 1 \\ 3x_1 + 3x_2 + 4x_3 = 2 \\ 2x_1 + 2x_2 + 3x_3 = 1 \end{cases}$$

Could you use part (c) of the previous problem to solve the system?

Problem 5.

We know that if A and B are nonsingular, then AB is also nonsingular. However, in general, the sum of two nonsingular matrices can be either nonsingular or singular.

- (1) Find examples of (2×2) matrices A and B such that A and B are nonsingular, but $A + B$ is singular.
- (2) Find examples of (2×2) matrices A and B such that A and B are nonsingular, and $A + B$ is also nonsingular, but $(A + B)^{-1} \neq A^{-1} + B^{-1}$.